SANDY CREEK HIGH SCHOOL
SUMMER REVIEW PACKET

For students entering ***A.P. CALCULUS***

By signing below, I acknowledge that I worked every problem on this assignment myself. While I was allowed to seek help from others, I did not copy someone else's work and submit it as if it was my own.

Printed Name: _____

Signature: _____ Date: _____

1. This packet is to be handed in to your Calculus teacher on the first day of the school year.
2. All work must be shown **in the packet**, next to the problem. Circle final answers.
3. You are expected to do this packet **without a calculator**.
4. Completion of this packet is worth one-half of a major test grade. It will be graded as follows:
   Packet completed with all work shown = 25 points
   Five randomly selected problems graded for accuracy @ 5 points each = 25 points
5. You will have a multiple choice Calculus Readiness Test on the first day of the school year that also will be worth one-half of a major test grade.
6. You will have a quiz on the first day of the school year over your understanding of the unit circle that will be worth one quiz grade.

***Note: In calculus, you ALWAYS use RADIAN mode. No answers should ever be in degrees.**

***Note: Decimal answers should be accurate to three decimal places. However, since this is a no calculator packet, you should provide exact answers. For example: Write $\sqrt{3}$, not 1.732.**

1. *Identify the following statements as true or false. If false, correct the statement or explain the error.*

Statement	True or False?	Correction/Explanation if false
$(x + 2)(x - 3) = 5$ $x + 2 = 5$ or $x - 3 = 5$ $x = 3$ or $x = 8$		
$\frac{3x-5}{6} = \frac{x-5}{2}$		
$\frac{14x-12w}{10+6y} = \frac{7x-6w}{5+3y}$		
$(x + 5)^2 = x^2 + 25$		
If $1 = \sqrt{2x+7} - \sqrt{x+3}$, then $1^2 = \left(\sqrt{2x+7}\right)^2 - \left(\sqrt{x+3}\right)^2$		
$\sqrt{x^2+49} = x+7$		
$\sqrt{49x^2} = 7x$		
$\sqrt{9} = \pm 3$		
If $x^2 = 9$, then $x = \pm 3$.		
$x^2 + x^2 = x^4$		
$\frac{x+5x^2}{x} = 5x$		
Solve for x: $x^2 = 4x$ I can divide both sides by x and I get $x = 4$.		
$f(x) = -x^2$ $f(-3) = -9$		

$f(x) = x^2$ $f(-3) = -9$		
$\left(3 \cdot w^3 \cdot x^4\right)^2 = 3w^6x^8$		
$\frac{y}{7x^{-1}} = 7xy$		
$2(3-g)(4+z) = (6-2g)(8+2z)$		
$2(5x) = (2 \cdot 5)(2 \cdot x)$		
ln(x) means the natural logarithm multiplied by x.		
sin(x) means the sine function multiplied by x.		
$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$		
$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$		
$\frac{(x-3)x - x^2(x+1)}{(x-3)} = x - x^2(x+1)$		
$\ln(x + e) = \ln(x) + \ln(e)$		
$2^{x+4} = 2^x + 2^4$		
$\cos^2 x = (\cos x)^2$		
$\cos^{-1} x = \sec x$		

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{3}$		
If I subtract $x - b$ from a, I get $a - x - b$.		
(5a)(5b) = 5(ab)		
$3(x + 6)^2 = (3x + 18)^2$		
$\frac{1}{3x}=\frac{1}{3}x$		
$\frac{x}{3}=\frac{1}{3}x$		

2. What is the point/slope form for the equation of a line?

3. Write the equation of the line whose slope is -4 and passes through (-3, 2).

4. Write the equation of the line parallel to $3x - 2y = 8$ that passes through the point (2, 5).

5. Write the equation of the line perpendicular to $3x - 2y = 8$ that passes through the point (2, 5).

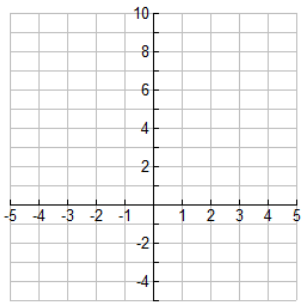
6. Find the average rate of change of $f(x) = x^2 + 6x - 10$ from $x = 2$ to $x = 4$.

7. Find $\frac{f(x+h)-f(x)}{h}$ for the given function f:

   a) $f(x)=9x+3$

   b) $f(x)=5-2x$

8. Graph $f(x)=\begin{cases} -1 & \text{if } x \le -2 \\ 1+x & \text{if } -2 < x \le 2 \\ 2x & \text{if } x > 2 \end{cases}$



Find the following:

f(-3) = _____ f(-2) = _____ f(-1) = _____ f(0) = _____ f(1) = _____ f(2) = _____ f(3) = _____

9. Let f(x) = $1 + \frac{1}{x}$ and g(x) = $\frac{3}{\sqrt{x}}$. Find f(g(x)).

10. Solve the system of equations: $\begin{aligned} 2x-3y &= -21 \\ 5x+6y &= 15 \end{aligned}$

11. Is the function even, odd, or neither. Justify your answer using f(x), f(-x) and –f(x).

a) $f(x)=\frac{x^4}{x^2-1}$

f(-x) = _____

-f(x) = _____

Even, odd, or neither? _____

b) $f(x) = x^3 + 2x^2 - 1$

f(-x) = _____

-f(x) = _____

Even, odd, or neither? _____

REMINDER NOTES FOR RATIONAL FUNCTIONS

Use the original problem for step 1.

1. Horizontal Asymptote / Slant Asymptote

- deg N < deg D — H.A. @ y = 0
- deg N = deg D — H.A. & y = ratio of leading coefficients
- deg N > deg D — no H. A.; Use long division to find slant asymp.

Factor the numerator and denominator, but do not cancel yet.

2. Domain

- Look at the denominator before you cancel and see what makes the bottom equal to zero.
- $\Re \neq$ those x-values

Now, cancel and simplify.

3. Holes

- If nothing cancels, there is not a hole.
- Set the canceled factor = 0. That is the x-coordinate of the hole.
- To get the y-coordinate, plug that x-value into the simplified function.

4. Vertical Asymptotes

- Set the denominator of the simplified function = 0 and find x.
- x = that value is the vertical asymptote

5. x-intercepts (zeros)

- Set the numerator of the simplified function = 0 and find x.
- (_____,0)

6. y-intercept

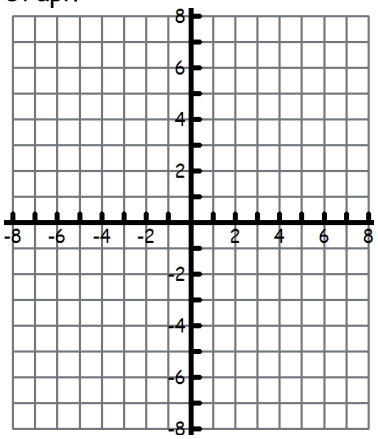
- Plug in zero for all the x's in the simplified function and evaluate it.
- (0,_____)

12. Identify all the necessary parts of the following rational functions and sketch it. Write "none" where applicable.

(a) $f(x) = \dfrac{3x^2 - 6x - 9}{x^3 - 9x}$

Horizontal Asymptote	
Slant Asymptote	
Hole(s)	
Vertical Asymptote(s)	
Domain	
x-intercepts	
y-intercepts	

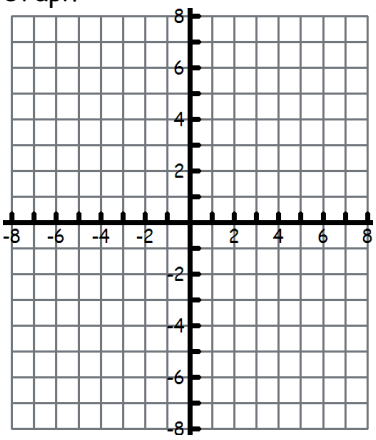
Graph



(b) $f(x) = \dfrac{2x^2 + 10x + 12}{x^2 + 3x + 2}$

Horizontal Asymptote	
Slant Asymptote	
Hole(s)	
Vertical Asymptote(s)	
Domain	
x-intercepts	
y-intercepts	

Graph



13. Simplify each expression.

a) $\dfrac{x^4 \cdot x^7}{x^6}$ b) $(s^4t^7)^3$ c) $\dfrac{x^2y^5}{(x^2)^4}$

14. Evaluate each expression.

a) $\left(27a^6\right)^{\frac{1}{3}}$ b) $27^{\frac{2}{3}}$ c) $\left(\dfrac{3}{4}\right)^{-1}$

15. Write each equation in exponential form.

a) $\log_{27} 3 = \dfrac{1}{3}$ b) $\log_{16} \dfrac{1}{4} = -\dfrac{1}{2}$

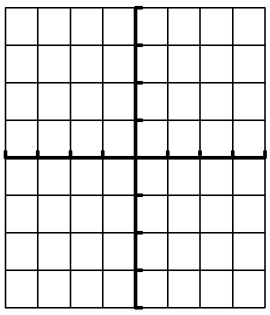
16. Write each equation in logarithmic form.

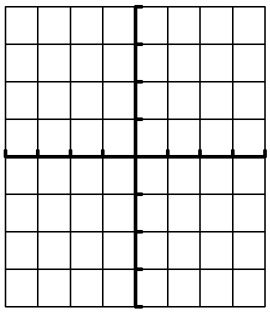
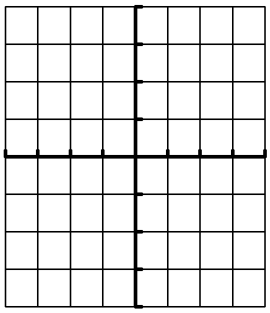
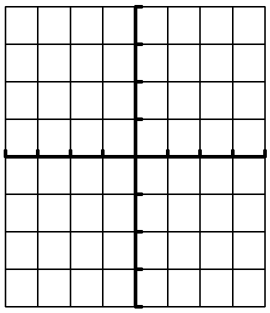
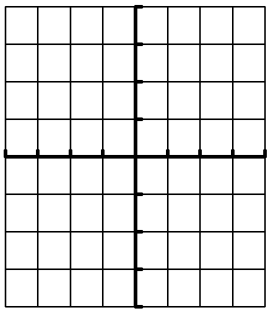
a) $10^2 = 100$ b) $e^0 = 1$

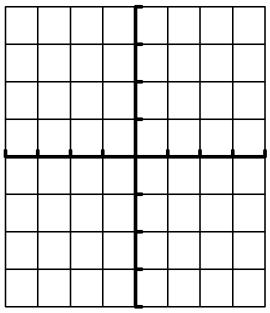
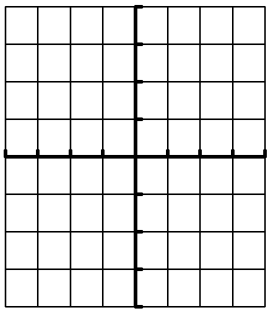
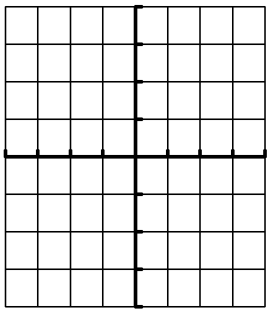
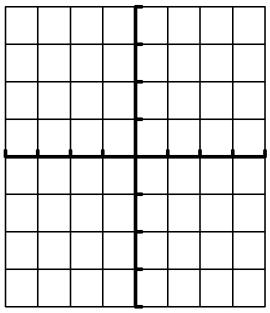
17. Use the Unit Circle to fill in the Chart. Give exact, reduced, rationalized answers.

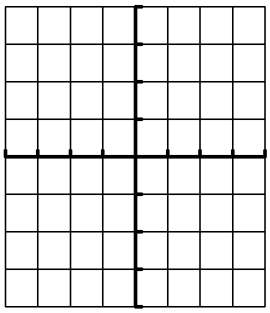
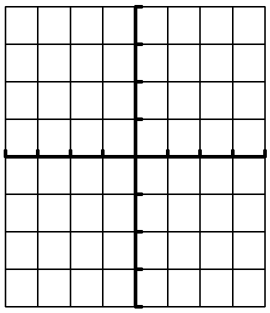
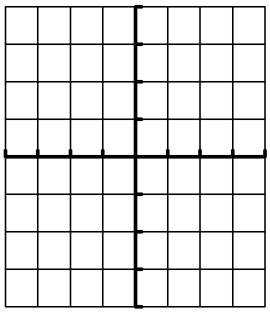
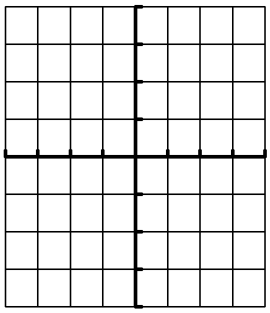
	sin (Θ)	cos (Θ)	tan (Θ)	csc (Θ)	sec (Θ)	cot (Θ)
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{2\pi}{3}$						
$\frac{3\pi}{4}$						
$\frac{5\pi}{6}$						
π						
$\frac{7\pi}{6}$						
$\frac{5\pi}{4}$						
$\frac{4\pi}{3}$						
$\frac{3\pi}{2}$						
$\frac{5\pi}{3}$						
$\frac{7\pi}{4}$						
$\frac{11\pi}{6}$						
2π						

18. Sketch a graph of the following functions and state the domain and range of each.

Function	Graph	Domain	Range
$f(x) = x$	 tick mark scale: x-axis_____ y-axis_____		

$f(x) = x^2$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = x^3$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \lvert x \rvert$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \sqrt{x}$	 tick mark scale: x-axis_____ y-axis_____		

$f(x)=\frac{1}{x}$	 tick mark scale: x-axis_____ y-axis_____		
$f(x)=\frac{1}{x^2}$	 tick mark scale: x-axis_____ y-axis_____		
$f(x)=\sin x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x)=\cos x$	 tick mark scale: x-axis_____ y-axis_____		

$f(x) = \tan x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = e^x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = \ln x$	 tick mark scale: x-axis_____ y-axis_____		
$f(x) = [x]$	 tick mark scale: x-axis_____ y-axis_____		

19. Solve each equation for x. Provide exact answers.

a) $\log_x 5 = -1/3$ b) $\log_{10}(2x+5) = \log_{10}(5x - 4)$ c) $\log_3(4x + 5) - \log_3(3 - 2x) = 2$

d) $4^{x^2-1} = 12$ e) $2^x = 3^{x+3}$ f) $\log_{\frac{3}{4}}\left(\frac{27}{64}\right) = x$

20. Assume that x is positive, use properties of logarithms to write the expression as a single logarithm.

a) $\frac{8}{5}\log_n 2x^2 + \frac{2}{3}\left(\log_n 2 + \log_n x^2\right)$ b) $\frac{1}{6}\log_4 x^{10} + \frac{1}{8}\log_4 x^4 - \frac{1}{12}\log_4 x$.

21. Assume that x is positive, use properties of logarithms to write the expression as a sum or difference of logarithms.

a) $\ln\left(\frac{\sqrt[5]{30}}{x^3 y^2}\right)$ b) $\ln\left(\sqrt[6]{\frac{\sqrt[3]{4x^9}}{z^8}}\right)$

22. Evaluate the expression without using a calculator.

a) $\left(3^2\right)^{\log_9 8}$ b) $e^{\ln\sqrt[4]{e}}$ c) $10^{\log\sqrt{43}}$ d) $\ln e^{931}$

23. a) Find the discriminant of $x^2 + 2x - 2 = 0$.

b) Describe the nature of the roots of the equation.

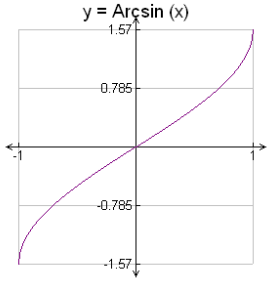
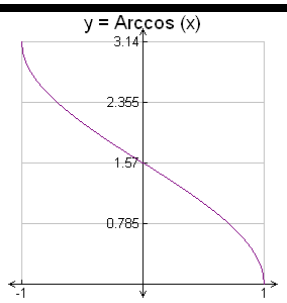
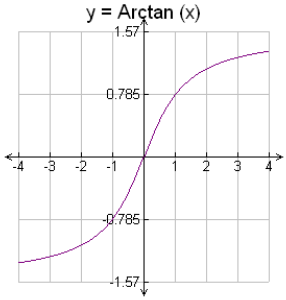
c) Solve the equation by using the Quadratic Formula.

24. a) Find the discriminant of $2x^2 + 3x + 5 = 0$

b) Describe the nature of the roots of the equation.

c) Solve the equation by using the Quadratic Formula.

Inverse Trigonometric Functions

Function	Graph	Domain	Range
y = arcsin (x) or $\sin^{-1}(x)$		$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
y = arccos (x) or $\cos^{-1}(x)$		$-1 \le x \le 1$	$0 \le y \le \pi$
y = arctan (x) or $\tan^{-1}(x)$		All reals	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

By restricting the domains and ranges, the inverse trig functions are actually functions.

25. Evaluate the following. Remember you are in RADIAN mode. Give exact answers and don't forget the restrictions as stated in the above table.

a) $\sin^{-1}(1) =$ _____ b) $\cos^{-1}(-1) =$ _____ c) $\tan^{-1}\left(-\sqrt{3}\right) =$ _____

d) $\tan^{-1}(-1) =$ _____ e) $\cos^{-1}\left(\frac{1}{2}\right) =$ _____ f) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ _____

g) $\tan\left(\cos^{-1}\frac{5}{13}\right) =$ _____ h) $\tan^{-1}(\cos(0)) =$ _____ i) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) =$ _____

j) $\cos\left(\cos^{-1}\frac{1}{2}\right) =$ _____ k) Find the algebraic expression equivalent to

$\sin\left(\tan^{-1}\left(\frac{2x+3}{5x}\right)\right)$. _____

26. Let $4x^2 + 4y^2 - 24x + 32y + 36 = 0$

a) Explain how you know this equation is a circle.

b) Complete the square to convert from general form to standard form. Show all steps.

c) State the center and radius. center: _____ radius:_____

d) State the domain and range. domain: _____ range: _____

27. Let $16x^2 + 25y^2 - 96x - 200y = -144$

a) Explain how you know that this is an ellipse by just looking at it.

b) Complete the square to convert from general form to standard form. Show all steps.

c) Graph the ellipse.

28. State the factoring rule for the following:

a) Difference of Two Squares: $a^2 - b^2$ = _____

b) Sum of Two Squares: $a^2 + b^2$ = _____

c) Difference of Two Cubes: $a^3 - b^3$ = _____

d) Sum of Two Cubes: $a^3 + b^3$ = _____

You should know the following identities:

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

1. **$\sin^2\theta + \cos^2\theta = 1$**
   ($\sin^2\theta = 1 - \cos^2\theta$)
   ($\cos^2\theta = 1 - \sin^2\theta$)

2. **$\tan^2\theta + 1 = \sec^2\theta$**
   ($\sec^2\theta - \tan^2\theta = 1$)
   ($\tan^2\theta = \sec^2\theta - 1$)

3. **$1 + \cot^2\theta = \csc^2\theta$**
   ($\cot^2\theta = \csc^2\theta - 1$)
   ($1 = \csc^2\theta - \cot^2\theta$)

Sum and Difference Identities

1. $\sin(A + B) = \sin(A)\cos(B) + \sin(B)\cos(A)$
2. $\sin(A - B) = \sin(A)\cos(B) - \sin(B)\cos(A)$
3. $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
4. $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
5. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Double Angle Identities

1. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
2. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
   $= 1 - 2\sin^2(\theta)$
   $= 2\cos^2(\theta) - 1$
3. $\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$